Initiation à la Recherche 2

Generalized Relations in Linguistics & Cognition

Jean-Baptiste Daval, Younesse Kaddar

Ecole Normale Supérieure Paris-Saclay

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Abstract

Generalized Relations in Linguistics & Cognition[1] (15 pages long)

Use of

- compact closed categories

as models for cognition and NLP.

Abstract

Generalized Relations in Linguistics & Cognition[1] (15 pages long)

Use of

- compact closed categories
- categories of generalized relations

as models for cognition and NLP.

Abstract

Distributional models of language

- Meaning of a word using occurrence statistics derived from corpus data
- graphical language for composite systems of abstract processes

Question: how to combine meanings of individual words to understand sentences?

$\longrightarrow$ Categorical compositional models of natural language

Pregroup grammar

Pregroup $(A, 1, \cdot, -^l, -^r, \leq)$

A monoid $(A, 1, \cdot)$ such that:

Contraction:

$$x^l \cdot x \leq 1 \qquad x \cdot x^r \leq 1$$

Expansion:

$$1 \leq x \cdot x^l \qquad 1 \leq x^r \cdot x$$

- x^r and x^l called left and right adjoints of x
- $\cdot$ and $\leq$ also written $\otimes$ and $\rightarrow$

Pregroup grammar: Grammatical sentences

- Set of words associated to types
- A sentence S that has type T is grammatical if $T \leq S$.

Example

John : N Mary : N met : $N^r \cdot S \cdot N^l$

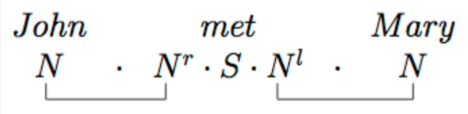


Figure 1: Example of a grammatical sentence

Autonomous categories

- Pregroups: Compositional features of natural language
- Meaning of words: Vector space models derived from co-occurrence statistics

Key point

Both pregroups and the category of finite dimensional real vector spaces are autonomous categories.

Autonomous categories & Compact closed categories

Monoidal

1. bifunctor $\otimes : \mathbf{C} \times \mathbf{C} \to \mathbf{C}$
2. identity object I
3. natural isomorphisms:
    - associator $\alpha_{A,B,C}$:

      $(A \otimes B) \otimes C \cong A \otimes (B \otimes C)$

    - unitors:

      $\lambda_A : I \otimes A \cong A$ and $\rho_A : A \otimes I \cong A$

4. + coherence conditions

Symmetric

1. Nat. isomorphism:

   $s_{AB} : A \otimes B \to B \otimes A$

2. associativity/unit coherences

Right/left duals

Dual objects:

unit $\eta_A : I \to A^* \otimes A$

counit $\varepsilon_A : A \otimes A^* \to I$

$$A \xrightarrow{\cong} A \otimes I \xrightarrow{A \otimes \eta} A \otimes (A^* \otimes A)$$
$$\xrightarrow{\cong} (A \otimes A^*) \otimes A \xrightarrow{\epsilon \otimes A} I \otimes A \xrightarrow{\cong} A$$

$$A^* \xrightarrow{\cong} I \otimes A^* \xrightarrow{\eta \otimes A^*} (A^* \otimes A) \otimes A^*$$
$$\xrightarrow{\cong} A^* \otimes (A \otimes A^*)$$
$$\xrightarrow{A^* \otimes \epsilon} A^* \otimes I \xrightarrow{\cong} A^*$$

Functorial semantics

Monoidal functor : functor that preserves the identity and the tensor product of the monoidal categories, while satisfying coherence laws

Functorial semantics

Monoidal functor

- from a pregroup describing grammatical structure
- to the category of finite dimensional vector spaces

It maps type reductions to linear maps $\Longrightarrow$ derive the meaning of a sentence from its parts

Goal

Constructing compact closed categories with good mathematical properties.

Hypergraph categories: Frobenius algebras

Frobenius algebra $(A, \mu, \eta, \delta, \varepsilon)$

An object A of C with four morphisms

- $\mu : A \otimes A \to A \qquad \eta : I \to A$
- $\delta : A \to A \otimes A \qquad \varepsilon : A \to I$

such that

- (A, μ, η) is a monoid object in C
- (A, δ, ε) is a comonoid object in C
- the following diagrams commute:

$$\begin{array}{ccc} A \otimes A & \xrightarrow{\delta \otimes A} & A \otimes A \otimes A \\ {\scriptstyle \mu}\downarrow & & \downarrow{\scriptstyle A \otimes \mu} \\ A & \xrightarrow[\delta]{} & A \otimes A \end{array} \qquad \text{and} \qquad \begin{array}{ccc} A \otimes A & \xrightarrow{A \otimes \delta} & A \otimes A \otimes A \\ {\scriptstyle \mu}\downarrow & & \downarrow{\scriptstyle \mu \otimes A} \\ A & \xrightarrow[\delta]{} & A \otimes A \end{array}$$

Hypergraph categories: definition

Hypergraph category

Symmetric monoidal category in which every object is equipped with a choice of special commutative Frobenius algebra, coherently with the monoidal structure.

From now on

All the compact closed categories will be hypergraph categories.

Quantale

A complete lattice Q with an associative multiplication operation $: Q \times Q \to Q$ satisfying a distributive property:

- $$x * (\bigvee_{i \in I} y_i) = \bigvee_{i \in I} (x * y_i)$$
- $$(\bigvee_{i \in I} y_i) * x = \bigvee_{i \in I} (y_i * x)$$

Rel(Q)

The Q-relations $A \times B \to Q$ form a compact closed monoidal category **Rel**(Q) (provided that Q is commutative).

Then: algebraic Q-relations over a general algebra $(\Sigma, E) \longrightarrow$ hypergraph category

Quantales

Spans:

- proof relevant relations in which $S_x(a, b)$ tells us that x witnesses that a and b are related.

In a computational linguistics or cognition:

- relations are derived from data
- exploit these proof witnesses to track evidence that certain relationships hold.

Authors & Workshops

Authors

Figure 2: Department of Computer Science, University of Oxford

- Bob Coecke
- Fabrizio Genovese
- Martha Lewis
- Dan Marsden

Bob Coecke

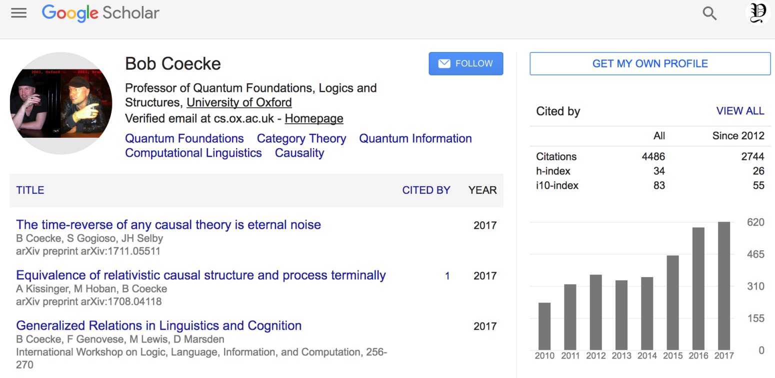


Figure 3: Bob Coecke on Google Scholar

Logic, category theory, categorical quantum mechanics, natural language processing.

WoLLIC

Workshop on Logic, Language, Information and Computation:
WoLLIC is an annual international forum on inter-disciplinary research involving formal logic, computing and programming theory, and natural language and reasoning. Each meeting includes invited talks and tutorials as well as contributed papers.
The conference is scientifically sponsored by the Association for Logic, Language and Information, the Association for Symbolic Logic, the European Association for Theoretical Computer Science and the European Association for Computer Science Logic.
Areas:

- Logics
- Language
- Computation
- Arithmetic & Foundations
- Applied Logic

Location: Brazil, France, Germany, US, Japan, ...

London

Table 1: Exemple articles

Paper	Authors	Area
Substructural Logics with a Reflexive Transitive Closure Modality	Sedlar	Logics
Coherent Diagrammatic Reasoning in Compositional Distributional Semantics	Wijnholds	Language
On the Computability of Graph Turing Machines	Ackerman, Freer	Computation

Other workshops

Symposium on Logical Foundations of Computer Science
This conference series provides an outlet for the fast-growing body of work in the logical foundations of computer science, e.g., areas of fundamental theoretical logic related to computer science.

Ouroboros: Formal Criteria of Self-Reference in Mathematics and Philosoph
The meeting is designed as a hybrid between winter school and research conference and will consist of plenary talks as well as introductory workshops which are intended to give insight into related areas of current research.

ACM/IEEE Symposium on Logic in Computer Science
The LICS Symposium is an annual international forum on theoretical and practical topics in computer science that relate to logic. LICS 2018 will be organized as part of the the Seventh Federated Logic Conference.

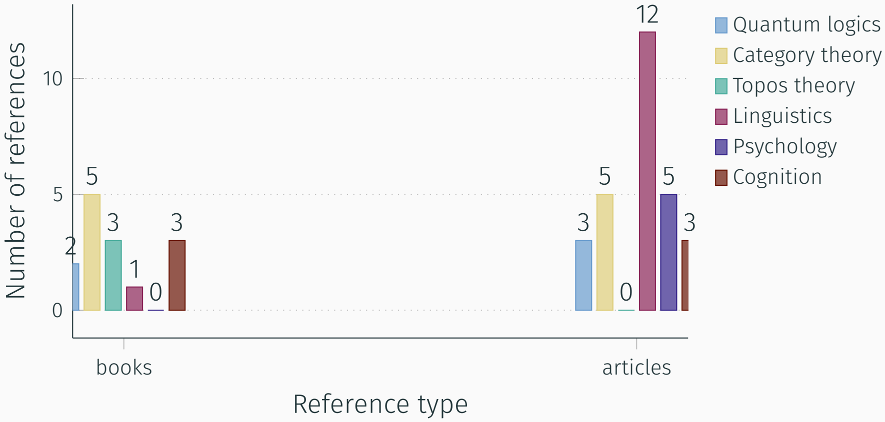
References

References

45 references, covering an array of different areas:

- Quantum logics & Categorical quantum mechanics
- Category theory
- Topos theory
- Computational linguistics & Natural language processing
- Psychology
- Cognitive science

Area bar chart



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A categorical semantics of quantum protocols

We study quantum information and computation from a novel point of view. Our approach is based on recasting the standard axiomatic presentation of quantum mechanics, due to von Neumann, at a more abstract level, of compact closed categories with biproducts. We show how the essential structures found in key quantum information protocols such as teleportation, logic-gate teleportation, and entanglement-swapping can be captured at this abstract level. This abstract and structural point of view opens up new possibilities for describing and reasoning about quantum systems.

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Conclusion

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